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Causality difficulties that can arise in modelling the reflection of a normally incident pulse from a lossy planar surface are studied theoretically. These difficulties can arise when the traditional approach to treating acoustic loss mechanisms (i.e., the generalization of lossless formulas via use of complex wavenumbers) is employed instead of using the proper viscoelastic relations. The problem caused by insufficient care being exercised in the choice of frequency dependence of the material properties of the lossy medium. In the present work, three models for the material properties of the lossy medium are considered. In all cases the phase speed in the material is assumed to be independent of frequency. Losses are assumed to be modellable with complex wavenumbers. The loss factor is chosen to be (1) frequency independent, (2) linearly dependent on frequency, and (3) quadratically dependent on frequency. For all three material models, exact analytical expressions for the reflected pressure are obtained. It is demonstrated that the requirements of causality are not satisfied for cases (2) and (3). These (over)

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Abstract (cont'd.)

results are interpreted mathematically in light of the appropriate Hilbert transform properties required for physically realizable signals. The analytical expressions presented herein for the reflected pressure are useful for investigators who numerically model echo-reduction systems.

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CASUALITY DIFFICULTIES WHICH CAN ARISE IN MODELLING THE REFLECTION OF A NORMALLY INCIDENT WAVE FROM A LOSSY PLANAR SURFACE

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CAUSALITY DIFFICULTIES WHICH CAN ARISE IN MODELLING THE REFLECTION OF A NORMALLY INCIDENT WAVE FROM A LOSSY PLANAR SURFACE

INTRODUCTION

The reflection of an acoustic wave normally incident on a lossy planar surface is of theoretical interest owing to its application to echo-reduction problems. Frequently, the loss mechanism of the material of the surface is treated theoretically by generalizing the solution for the lossless case via introduction of complex wavenumbers, although this treatment is only approximately correct [1]. If this approach is used, and insufficient care is exercised in the choice of the frequency dependence of the material properties of the substance comprising the lossy surface, then the causality condition may not be satisfied.

This possibility is investigated in the present work by analytically computing the reflected pressure for three simple material models. The material of the planar surface is modelled to have a frequency-independent phase speed in all three cases. The lossy nature of the surface is chosen to depend on frequency such that it (1) is constant, (2) varies linearly with frequency, or (3) varies quadratically with frequency. The approach to the reflection problem is to decompose the incident pressure field into its Fourier plane-wave components. The interaction of each of these components with the surface is then evaluated using the well-known result for the reflection of a plane wave from a semi-infinite material region (allowing the wave velocity in the material and its associated wave vector to be complex). It will be shown that when this approach is applied to the three material models described above, only case (1) (frequency-independent loss) results in a reflected pressure with appropriate causality properties. Hence, caution must be exercised if model (2) or (3) is applied to situations in which the attendant causality difficulties would be unacceptable. (Such causality difficulties can arise when a truncated Fourier solution is used, due to Gibb's phenomenon. This is not the case in what follows, however, since the required integrals are evaluated exactly by analytical means.)

This problem has been considered previously [2]; however, approximations were used to evaluate the relevant integrals. These approximations were not necessary in the present research. Also, although it is clear from Fig. 3 of Ref. 2 that causality difficulties arise from these models, no specific discussion of this is given in that reference. The purpose of the present work is to identify these causality difficulties, explain why they arise, and discuss their resolution.

PRELIMINARY CONSIDERATIONS

The normally incident pressure field is assumed to be of the form

$$p_i(t) = \begin{Bmatrix} 0 \\ p_o\ (1 - e^{-\gamma t})\sin(\omega_o t) \\ 0 \end{Bmatrix} \quad \begin{matrix} t<0 \\ \tau>t>0 \\ t>\tau \end{matrix} \quad . \tag{1}$$

The exact solution for the reflected field is quite complicated to obtain, involving boundary conditions that not only specify the usual displacements and stresses but also involve the acoustic temperature [3]. However, there is a large body of empirical evidence that demonstrates that acoustic losses in materials can be modelled fairly accurately in many cases by simply generalizing the solution for the lossless case by suitably allowing the wavenumber (and the wave speed) to be complex. This latter approach is adopted here. Thus, each of the plane-wave components of the pressure field within the lossy material itself is assumed to be representable by functions of the form

$$p_T(\omega) = p_{To} e^{i(-\tilde{k}x + \omega t)} \quad , \tag{2}$$

where $\tilde{k} = \omega/(c_{ph}) - i\alpha$ is the complex wavenumber. Here, ω is the angular frequency, c_{ph} is the phase speed in the material, and α is the loss factor. The Fourier components of the incident field are computed from $p_i(t)$ using

$$A_i(\omega) = \frac{1}{2\pi} \int_0^{\tau} p_i(t) e^{-i\omega t} dt \ ; \tag{3}$$

and, due to the reality of the incident field, the inverse Fourier transform may be written

$$p_i(t) = 2\mathrm{Re} \int_0^{\infty} A_i(\omega) e^{i\omega t} d\omega \ . \tag{4}$$

The symbol $r(\omega)$ is used to represent the (complex) reflection coefficient, where the well-known result [4], generalized using a complex velocity, is

$$r(\omega) = \frac{\rho_i \tilde{c}_i - \rho_o c_o}{\rho_i \tilde{c}_i + \rho_o c_o} \quad . \tag{5}$$

Here, ρ_o and c_o are the density and phase speed of the fluid (assumed lossless) in contact with the lossy medium. Also, ρ_i and $\tilde{c}_i$ are the density and complex sound velocity in the lossy medium itself, with $\tilde{c}_i = \omega/\tilde{k}$. The reflected pressure is computed from $r(\omega)$ using the formula

$$p_r(t) = 2\mathrm{Re}\int_0^\infty r(\omega)A_i(\omega)e^{i\omega t}d\omega . \tag{6}$$

ANALYTICAL COMPUTATION OF THE REFLECTED PRESSURE

Before proceeding further, it is helpful to note that $r(\omega)$ can be put into a convenient form with the aid of the following definition:

$$a_\pm = \rho_i \pm \frac{\rho_o c_o}{c_{ph}} . \tag{7}$$

Using this, $r(\omega)$ becomes

$$r(\omega) = \frac{a_-\omega + i\alpha\ \rho_o c_o}{a_+\omega - i\alpha\ \rho_o c_o} . \tag{8}$$

The reflected pressure will next be evaluated analytically for each of the three simple models of interest.

Constant Loss Factor

When Eq. (8) is substituted into Eq. (6) (with α = constant), the resulting integral can be put into a form amenable to the use of standard integral tables [5] via partial fraction analysis. However, the relevent tabular expressions involve functions possessing a branch cut, which can be particularly troublesome if caution is not exercised. Following is a list of integrals that result from a careful analysis of this branch cut:

$$\int_0^\infty \frac{\sin(ax)dx}{x+\beta} = \mathrm{ci}(a\beta)\sin(a\beta) - \cos(a\beta)\mathrm{si}(a\beta) \tag{9}$$

where $\mathrm{Re}(\beta)>o$ and $\mathrm{Im}\ (\beta)>0$, but not simultaneously = 0. Also, a is real and > 0.

$$\int_0^\infty \frac{\cos(ax)dx}{x+\beta} = -\sin(a\beta)\mathrm{si}(a\beta)-\cos(a\beta)\mathrm{ci}(a\beta) \tag{10}$$

where $\mathrm{Re}\beta>o$ and $\mathrm{Im}\beta>0$, but not simultaneously = 0. Again, a is real and > 0.

$$\int_0^\infty \frac{\sin(ax)dx}{\gamma - x} = [ci(a\gamma) - i\pi]\sin(a\gamma) - \cos(a\gamma)[si(a\gamma) + \pi], \tag{11}$$

where $Re(\gamma)>0$, and $Im\ \gamma \neq 0$. (If $Im\ \gamma = 0$, $i\pi$ term is dropped.) Again, a is real and >0.

$$\int_0^\infty \frac{\cos(ax)dx}{\gamma - x} = \sin(a\gamma)[si(a\gamma) + \pi] + \cos(a\gamma)[ci(a\gamma) - i\pi], \tag{12}$$

where $Re(\gamma)>0$, and $Im\ \gamma \neq 0$ (If $Im\ \gamma = 0$, $i\pi$ term is dropped.) Again, a is real and >0.

In Eqs. (9) thru (12), ci and si are the cosine integral and sine integral, respectively [6]. Using these results, analysis of the required integral is straightforward, though tedious. The result is:

$$p_r(t) = p_o\ Re\left\{ie^{i\omega_o t}\left[\left(\frac{-a_-\omega_o - i\alpha\rho_o c_o}{a_+\omega_o - i\alpha\rho_o c_o}\right) + \left(\frac{a_-(\omega_o+i\gamma) + i\alpha\rho_o c_o}{a_+(\omega_o+i\gamma) - i\alpha\rho_o c_o}\right)e^{-\gamma t}\right]\right\}$$

$$-\left(\frac{1}{a_+}\right) Im\left\{\left[\frac{i\alpha\rho_o c_o\left(1 + \frac{a_-}{a_+}\right)}{\omega_o - \left(\frac{i\alpha\rho_o c_o}{a_+}\right)}\right] + \left[\frac{i\alpha\rho_o c_o\left(1 + \frac{a_-}{a_+}\right)}{-\omega_o + i\gamma - \left(\frac{i\alpha\rho_o c_o}{a_+}\right)}\right]\right\}e^{-\alpha\rho_o c_o t/a_+}$$

$$\tau > t > o\ . \tag{13}$$

It should be noted that in the limit as $\alpha\to0$, Eq. (13) becomes the well-known lossless result $p_r(t) = (a_-/a_+)\ p_o\sin(\omega_o t)\ (1-e^{-\gamma t})$.

Loss Factor Proportional to Frequency

Next, the function α is allowed to assume the form $\alpha = \alpha_o(\omega/\omega_o)$. It should be noted from Eq. (8) that, in this case, the reflection coefficient $r(\omega)$ is frequency-independent. Hence, this function may simply be factored out from the integral of Eq. (6). [In what follows, the symbol r_o is used to represent the complex constant $r(\omega)$.] However, the resulting integral does not simply reproduce the incident pressure $p_i(t)$ [compare Eq. (4)]. This is because the <u>imaginary</u> part of the integral, which does not contribute to the computation of $p_i(t)$, <u>does</u> contribute to the reflected pressure due to the

effect of the complex (constant) reflection coefficient. It is also interesting to note that since $r(\omega) = r_o$ is constant in this case, it follows from Eq. (5) that the complex sound speed $\tilde{c}_i$ is also a constant.

The integrals arising in this case can still be evaluated using Eqs. (9) thru (12). Rather than directly displaying the reflected pressure in this case, it is somewhat more convenient to introduce a new quantity I, where

$$I = 2 \int_0^\infty A_i(\omega) e^{i\omega t} d\omega \ . \tag{14}$$

It should be noted that this is also the integral appearing in Eq. (4). The reflected pressure then becomes:

$$p_r(t) = \mathrm{Re}(r_o I)$$

$$\tau \geq t \geq 0 \ , \tag{15}$$

where

$$\begin{aligned} I = p_o \Bigg[& \frac{i}{\pi} \left\{ \mathrm{Im}\ e^{i\omega_o t} \left[ci(\omega_o t) - i si(\omega_o t) \right] \right\} \\ & - i\, e^{i\omega_o t} \\ & - \frac{i}{\pi}\ \mathrm{Im}\left(e^{i\omega_o t} \left| ci[\omega_o(\tau-t)] + i si[\omega_o(\tau-t)] \right| \right) \\ & + e^{-\gamma t} \left(\frac{i}{\pi}\ \mathrm{Im}\left\{ e^{i\omega_o t} \left[ci(\omega_o + i\gamma)(\tau-t) \right.\right.\right. \\ & \left.\left. + i si(\omega_o + i\gamma)(\tau-t) \right] \right\} \\ & + \frac{i}{\pi}\ \mathrm{Im}\left\{ e^{-i\omega_o t} \left[ci(\omega_o - i\gamma)t + i si(\omega_o - i\gamma)t \right] \right\} \\ & \left. + i e^{i\omega_o t} \right) \Bigg] \ . \end{aligned} \tag{16}$$

The real part of Eq. (16) is equal to $p_o(1-e^{-\gamma t})\sin(\omega_o t)$, as is expected based on its relationship to Eq. (4), the Fourier representation of the incident field.

Loss Factor Proportional to Square of Frequency

Finally, the function α is allowed to assume the form $\alpha = \alpha_o(\omega/\omega_o)^2$. When this is substituted into Eq. (8), we obtain an expression for $r(\omega)$ very similar to that obtained for the case α = constant (in the present case, a factor of ω is present in the imaginary parts of the numerator and the denominator of $r(\omega)$, whereas in the case α = constant, a factor of ω appeared in the real parts). Hence, a very similar partial fraction decomposition is possible, and Eqs. (9) thru (12) again suffice to evaluate the resultant integrals. Before displaying the final expression for the reflected pressure, a few definitions are helpful:

$$\delta_1 = \frac{a_- + a_+}{\dfrac{\omega_o + i\, a_+ \omega_o^2}{\alpha_o \rho_o c_o}}$$

$$\delta_2 = \frac{a_- + i\,\dfrac{\alpha_o \rho_o c_o}{\omega_o}}{a_+ - i\,\dfrac{\alpha_o \rho_o c_o}{\omega_o}}$$

$$\delta_3 = \frac{a_- + a_+}{-\omega_o + i\gamma + i\,\omega_o^2\,\dfrac{a_+}{\alpha_o \rho_o c_o}}$$

$$\delta_4 = \frac{a_- + i\,\dfrac{\alpha_o \rho_o c_o}{\omega_o^2}(\omega_o + i\gamma)}{a_+ - i\,\dfrac{\alpha_o \rho_o c_o}{\omega_o^2}(\omega_o + i\gamma)} .$$

In terms of these quantities, the reflected pressure becomes:

$$p_r(t) = p_o \left\{ -\mathrm{Re}\left(i\delta_2 e^{i\omega_o t}\right) + e^{-\gamma t}\ \mathrm{Re}\left(i\delta_4 i e^{i\omega_o t}\right) \right.$$

$$\left. - \frac{\omega_o^{\,2}}{\alpha_o \rho_o c_o}\ \frac{-\omega_o^{\,2} a_+(\tau - t)}{e\ \alpha_o \rho_o c_o} \left[\mathrm{Re}\left(\delta_1 e^{i\omega_o \tau}\right) - e^{-\gamma\tau}\ \mathrm{Re}\left(\delta_3 e^{i\omega_o \tau}\right)\right]\right\}$$

$$\tau \geqslant t \geqslant 0\ . \qquad (17)$$

CAUSALITY

Consider next the consequences of the above results [Eqs. (13), (15), and (17)] in regard to causality. Although no difficulties arise for Eq. (13) (the case where α = constant), this is not so for Eqs. (15) and (17) (where α is proportional to frequency and proportional to the square of the frequency, respectively). The most obvious way to see that there are difficulties in the latter cases is to note that the results are dependent on the pulse duration τ, although the results presented are valid for times less than τ. This is clearly inconsistent with causality requirements, since it means that the "end" of the incident pulse influences the reflected pressure prior to its arrival at the reflecting surface. Although not as obvious, careful analysis of these expressions also shows that when $\alpha_o \neq 0$ then $p_r(t) \neq 0$ at $t = 0$. Hence, once again, these expressions contradict the causality requirement that no reflected wave should arise prior to the arrival of the incident wave.

There is a simple alternative way to see that causality difficulties will arise for these models. This is by computing the impulse response of the given system [i.e., by computing the reflected pressure that would arise for an incident field of the form $p_i(t) = p_o\delta(t)$, where $\delta(t)$ is the Dirac delta function]. This is done by computing the inverse Fourier transform of the function $r(\omega)$. As an example, consider the model in which α is linearly dependent on frequency. Recall that, in this case, $r(\omega)$ is a complex constant (chosen to be r_o). Thus

$$r(t) \equiv F^{-1}\ [r(\omega)] = 2\mathrm{Re}\left(r_o \int_0^\infty e^{i\omega t} d\omega\right), \qquad (18)$$

or

$$r(t) = 2\mathrm{Re}\{r_o[\pi\delta(t) + \tfrac{i}{t}]\}. \qquad (19)$$

Here, F^{-1} refers to the inverse Fourier transform. From Eq. (19) it is clear that when $\mathrm{Im}(r_o) \neq 0$, then $r(t) \neq 0$ for $t < 0$. Hence, there is a non-zero response prior to $t = 0$ to an impulse at $t = 0$.

Although the above conclusions may be disturbing, they are not unreasonable for the following reason: In order for a signal to be physically realizable (i.e., causal), it must conform to specific mathematical conditions. In fact, the real and imaginary parts of the frequency spectrum of the signal must be Hilbert transforms of each other [7,8]. In view of this rather stringent mathematical requirement, it is unlikely that the reflected pressure $p_r(t)$ arising from the interaction of a wave with a material of arbitrarily selected material properties will be consistent with the required transforms. In fact, in retrospect, it is remarkable that the case α = constant does not result in causality difficulties. The most probable rectification of the problems arising in the latter two cases is to select an appropriately frequency-dependent material phase speed (i.e., it must be that the only functional form for α which is consistent with c_{ph} = constant is α = constant as well). It is not intended here to imply that such approximations as c_{ph} = constant and α proportional to ω^2 are not reasonable for many materials. It is only intended to point out that one consequence of using these approximate expressions is the breakdown of the causality condition. (For guidance in the proper selection of the frequency dependence of the real and imaginary parts of material properties, see Refs. 1 and 8.)

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6. See definitions for si and ci in, for example, M. Arbamowitz and I. A. Stegun, Handbook of Mathematical Functions (Dover, New York, 1972).

7. R. Bracewell, The Fourier Transform and its Applications (McGraw-Hill, New York, 1965) p. 272.

8. There is also a required relationship between the mathematical form for the loss function and the sound speed. This relationship is: $\omega/c(\omega) = \omega/b + [\alpha(\omega)]$, where $c(\omega)$ is the sound speed, b is a constant, and $[\alpha(\omega)]$ denotes the Hilbert transform of the loss function. See J. E. White, Underground Sound (Elsevier, New York, 1983) p. 134, Eq. (4-66).

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